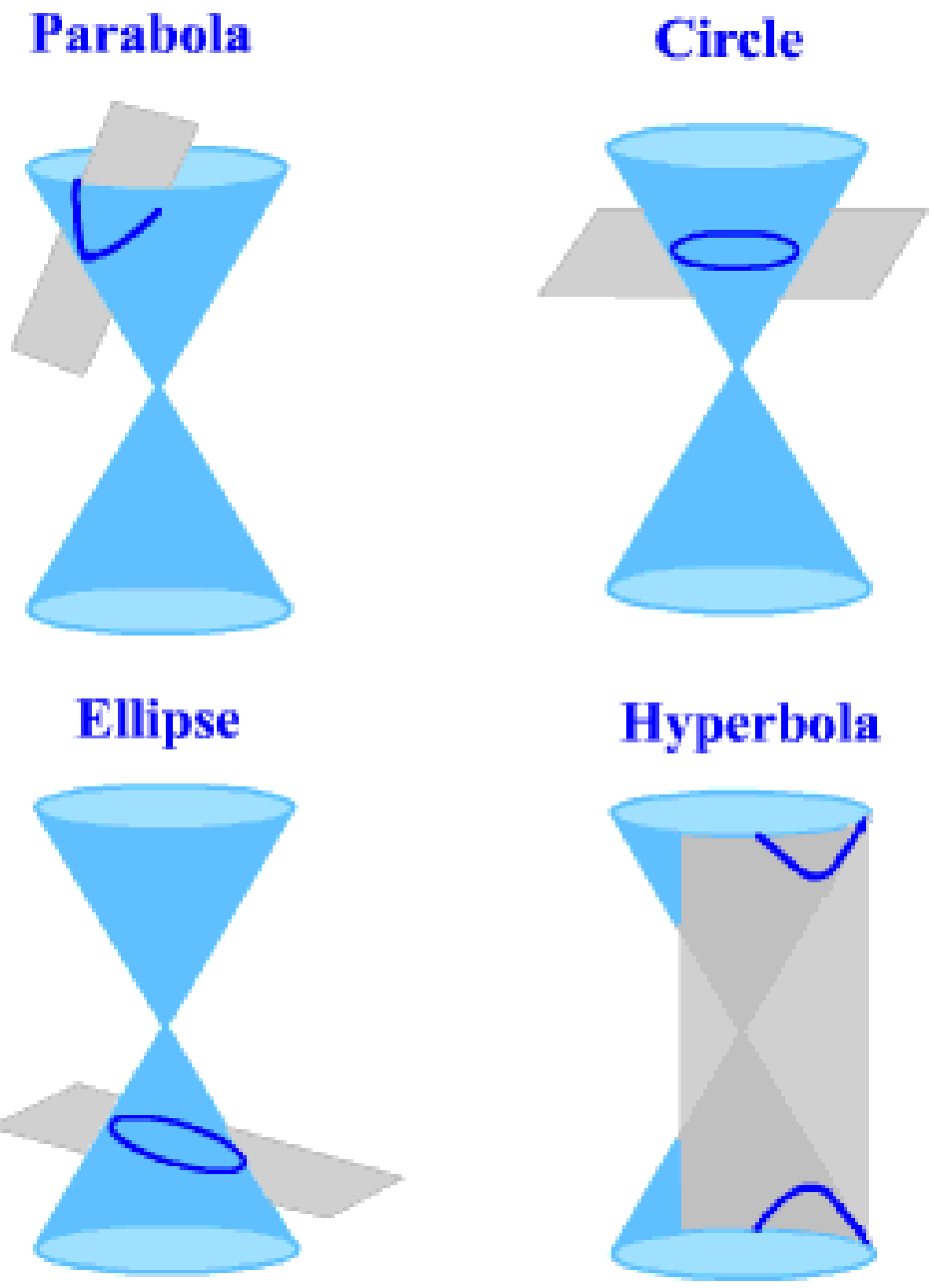
AS 90639 CONIC SECTIONS – sketching graphs of conic sections and writing equations related to conic sections.

A conic section is the intersection of a plane and an upright circular cone. The four basic types of conics are:

- The circle
- The ellipse
- The parabola
- The hyperbola



A) CIRCLE

The general equations of a circle are:

$x^2 + dx + y^2 + ey + f = 0$

$x^2 + y^2 = r^2$ centre (0, 0), radius = r

$(x - a)^2 + (y - b)^2 = r^2$ centre (a, b), radius = r

COMPLETING THE SQUARE FOR A CIRCLE

Sometimes the equation for a circle is given in the 1st form above where it is not possible to identify just by looking at it whether the equation is a circle or not, and if it is, what the centre and radius are. Therefore we separate out the x terms and the y terms, then complete the square for each.

Example:

1) Complete the square for the equation $x^2 - 4x + y^2 + 6y = 12$ to obtain the coordinates of the centre of the circle and its radius.

$x^2 - 4x + y^2 + 6y = 12$
$\left[x^2 - 4x + (\frac{-4}{2})^2\right] + \left[y^2 + 6y + \left(\frac{6}{2}\right)^2\right] = 12 + \left(\frac{-4}{2}\right)^2 + \left(\frac{6}{2}\right)^2$
$[x^2 - 4x + 4] + [y^2 + 6y + 9] = 12 + 4 + 9$
$(x - 2)^2 + (y + 3)^2 = 25$
$(x - 2)^2 + (y + 3)^2 = 5^2$

So centre is (2, -3) and radius is 5.

2) Find the centre and the radius of the circle $x^2 + y^2 - 8x - 16y + 55 = 0$.

$x^2 + y^2 - 8x - 16y + 55 = 0$
$x^2 - 8x + y^2 - 16y = -55$
$\left[x^2 - 8x + (\frac{-8}{2})^2\right] + \left[y^2 - 16y + (\frac{-16}{2})^2\right] = -55 + (\frac{-8}{2})^2 + (\frac{-16}{2})^2$
$[x^2 - 8x + 16] + [y^2 - 16y + 64] = -55 + 16 + 64$
$(x - 4)^2 + (y - 8)^2 = 25$
$(x - 4)^2 + (y - 8)^2 = 5^2$

Worksheet
Delta Ex 37.1 pg 358 Q1, 2
Extension Ex 37.1 Q3 onwards (Q9 refer to example 4 in textbook)